

# Lunatic Fringe:

## The Science and Math of the Moon

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webpage with images and links:

[http://www.astro.uiuc.edu/~bdfields/mste\\_aims07.html](http://www.astro.uiuc.edu/~bdfields/mste_aims07.html)

- ▶ The geometric Moon:
  - Lunar phases and the Moon's orbit
  - eclipses of the Sun and Moon
  
- ▶ The dynamic Moon:  
gravity, tides, the receding Moon and lengthening day
  
- ▶ The statistical Moon:  
counting craters on the Moon for solar system archaeology
  
- ▶ The physical Moon  
first-person accounts, lessons from moonrocks

## Introductions

Please allow me to introduce myself...

*Q: anybody teach any astro?*

*Q: if so, what? do you use math?*

Today's schtick based on my conviction:

**Astronomy** and **Math/Science teaching**  
are **two great tastes** that **taste great together!**

Astronomy can serve math & science by:  
illustrating, motivating, and enriching subjects you are  
*already teaching*

2 Today's strategy: I'll provide the Astronomy  
but you know best how build your lessons!

# The Geometric Moon: Orbit, Phases, Eclipses

# The Moon's Motion on the Sky

as seen by earthling, Moon moves in the sky  
several patterns emerge

## daily motion:

Moon rises in the eastern horizon,  
sets in the westerns horizon  
period  $P_{\text{horizon}} = 1 \text{ day}$

## monthly (“Moonthly”) motion:

Moon moves among the stars *Q: what does this mean?*  
path is circle—in fact, a great circle  
travels in Zodiac (same constellations as Sun)  
period  $P_{\text{thru stars}} = 1 \text{ month}$

↳

*Q: what do these patterns tell us about the Moon's motion in 3-D space?*

## Lessons from Lunar Motions

- Daily Motion

Moon rises in E, sets in W; Sun, stars do same  
Earth-centered (“geocentric” or egocentric!) interpretation

→ we are fixed, all orbit us once daily!

not crazy! ancient Greeks believed this!

Sun-centered (“heliocentric” or democratic) interpretation

**our Earth spins** → earthlings see spinning sky

infer  $P_{\text{spin,Earth}} = 1 \text{ day!}$

- Monthly Motion

on 2-D sky: Moon moves among (and in front of!) stars

in 3-D space: stars are (more or less) fixed

→ Moon and Earth must move relative to each other!

→ **Moon orbits Earth**, in a plane,

with a nearly circular path

infer  $P_{\text{orbit,Moon}} = 1 \text{ day!}$

## Interlude: The Flat Sky

The motion of the Moon is one example of a general problem in astronomy (and all science!)  
have to connect

(a) what you can actually *observe/measure: data*

(b) with what is “really” going on—*models/theory*

In astronomy: observe objects in sky  
can measure shapes, positions on sky

but sky gives *no* direct information about *distance*

→ observed sky flattens the 3-dimensional arrangements

o down to 2-D sphere projection: “cosmic roadkill”

Geometry problem: have to always go between

- ▷ observed 2-D sky view
- ▷ underlying 3-D space distribution

For the Moon

in 3-D space: a sphere orbiting Earth,

itself spinning on its own axis, and orbiting Sun

on 2-D sky as seen by earthling: a circular disk, none/some/all illuminated

Other “flattening” issues in astronomy...

Exercise in geometry, spatial visualization.

## A Sense of Scale

How do sizes of the Earth and Moon compare?

If Earth represented by globe  $\approx$  basketball size

*What would correctly represent the Moon's size?*

i.e., when is  $r_{\text{ball}}/r_{\text{globe}} = R_{\text{Moon}}/R_{\text{Earth}}$ ?

- (a) another basketball
- (b) tennis ball
- (c) golfball
- (d) marble

How does the orbit compare to the object sizes?

If Earth represented by globe and Moon by ball

*Where does the "moonball" need to travel?*

- (a) arm's length
- (b) front row
- (c) middle of room
- (d) back of room
- (e) outside of the building

Earth radius:  $R_{\text{Earth}} \approx 6400 \text{ km}$

Moon:  $R_{\text{Moon}}/R_{\text{Earth}} \approx 1/4$

★ if Earth  $\rightarrow$  basketball, Moon  $\approx$  tennis ball

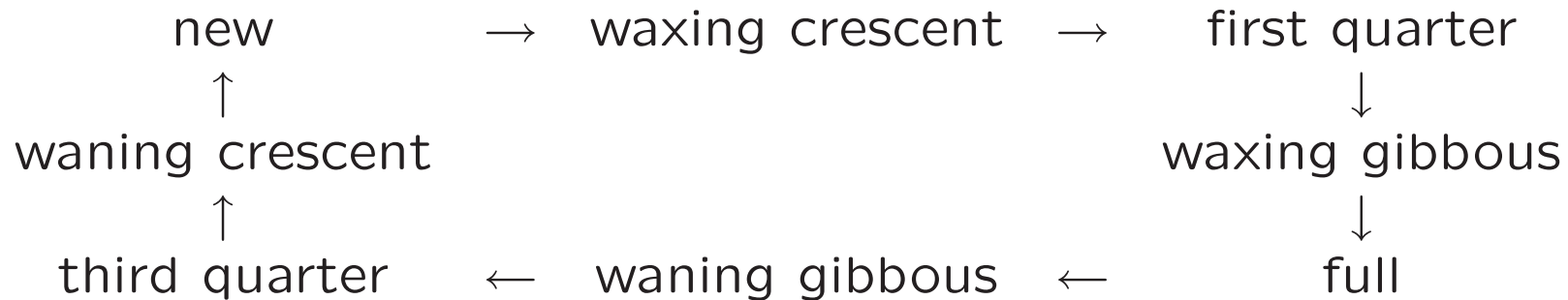
Moon orbit:  $d_{\text{Moon}} \approx 60R_{\text{Earth}}$

★ if Earth  $\rightarrow$  basketball, Moon  $\approx$  10 meter  $\approx$  30 ft away!

surprisingly far! key to remember regarding phases

# Phases of the Moon

another monthly lunar pattern: cycle of phases



www: lunar phase images

*Q: what is basic physical origin of phases?*

*Why do we sometimes see only part of the Moon illuminated?*

*What does the monthly period imply?*

phases simple but beautiful

basic effect: **see illuminated moon from different angles**

phases **not** due to Earth blocking sunlight

i.e., phases are not eclipses!

## **Phases Take 1: a gut feeling for the effect**

*Demo: Be the Earth and Moon yourself!*

- use flashlight/bright lightbulb for Sun
- make fist (or use ball) for Moon
- Earth = your head!

spin slowly → Moon orbit, watch phases on fist/ball/Moon change

## Phases Take 2: analyzing the geometry

*diagram: top view, sky views*

excellent exercise in translating situation in 3-D space  
to 2-D projection on sky

for each Sun-Earth-Moon position, ask:

*Q: how much of Moon's surface is illuminated by the Sun?*

*Q: how much of the illuminated portion can we see from Earth?*

*Q: what does this look like in the sky?*

Hint: to check you answers, redo flashlight Sun experiment

Note: each phase rises and sets at a specific time of day

*Q: when 1st Quarter moonrise?*

my suggestion: do it! practice!

when moon up, point to moon, sun, look at angle!

# Eclipses

**lunar eclipse:** moon in earth's shadow

*diagram: Sun, Earth, Moon*

www: lunar eclipses

note: can still see Moon even when totally in Earth's shadow!

appears much dimmer, and red

*Q: what's going on? why the red color?*

note that *direct* sunlight is totally blocked

so light must be indirect, in fact:

scattered light from earth's atm.

red b/c blue is scattered more strongly, so only red is left

in other words:

glow is from all the sunrises and sunsets on Earth!

**solar eclipse**: observer in moon's shadow

Note: Earth larger than Moon

*Q: what does this immediately imply for solar eclipses?*

since Moon smaller than earth, whole earth cannot be in shadow  
in fact, only a small region,  $\sim 100$  mi, at a given time  
“eclipse path”

interesting coincidence: Moon and Sun *appear* to have  
almost identical *angular* size on the sky  
i.e., one can “cover” the other nearly exactly  
*and what's more*: Moon's distance changes (not circular orbit)  
www: moon perigee/apogee comparison

together this means: two kinds of solar eclipses

www: annular eclipse

www: total eclipse

www: looking back on Earth's shadow

16 Note: there is *not* a solar eclipse every month!

Q: *why? What does this imply for lunar motion in 3-D?*

if Moon's orbit plane around Earth  
were same as Earth's around the Sun (i.e., the ecliptic)  
then *would* have eclipses monthly

the fact that we *don't* means that  
the two orbits are *not* coplanar!

★ moon's orbit plane slightly tilted w.r.t. ecliptic  
so moon is typically below or above ecliptic

**Transp:** *eclipse diagram*

only eclipse when Moon orbit crosses ecliptic plane  
happens twice a year → “season of eclipses”

## A Spinning Moon?

www: lunation animation: always same face!

www: far side

Always same side faces us!

*demo: lunar globe*

*Q: in one orbit period, is there spin?*

*Q: how many periods?*

Moon has  $P_{\text{orbit}} = P_{\text{spin}}$  exactly!  
“corotation”

Sounds too good to be true!  
Better be a good reason for this!

...and there is...

ENOUGH ALREADY—TAKE A BREAK!

## The Dynamic Moon: Gravity, Tides, and the Lengthening Day

## Earth–Moon System: Tides

www: high/low comparison photo

ocean tides are due (mostly) to Moon

*Q: why “mostly”? Other actor(s)?*

ocean tides are **periodic**

*Q: what is tide period: high to high/low to low?*

www: tide data

*Q: how does the Moon raise tides on Earth?*

*Why the observed periodicity?*

## Tide Warmup: Gravity in a Nutshell

Isaac Newton:

gravity intimately linked to *mass*

- all objects with mass exert force: gravity
- all objects with mass feel gravity forces  
from all other massive objects

in fact: more mass → more gravity

but: gravity forces *decrease* as distance *increases*

→ *inverse* relationship with distance

General rule: two objects of mass  $m_1$ ,  $m_2$   
separated by distance  $d$

each feel attractive gravity force  $F$  due to the other

- $F \propto m_1$
- $F \propto m_2$
- $F \propto 1/d^2$

together:

$$F_{\text{grav}} \propto \frac{m_1 m_2}{d^2} \quad (1)$$

“inverse square law”

→ double distance:  $F_{\text{grav}}$  is 4 times smaller!

→ to lose weight, go upstairs!

*Q: effect of Earth's gravity on Moon?*

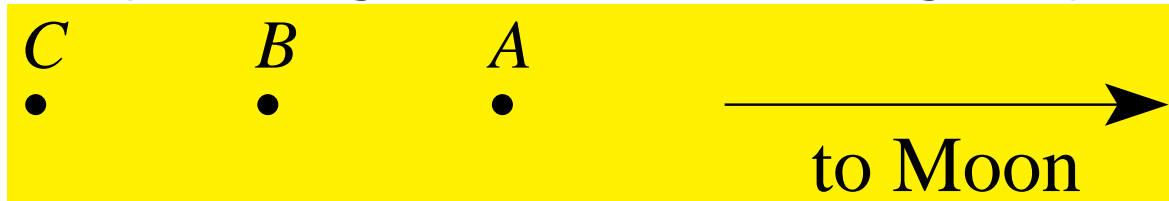
*Q: effect of Moon's gravity on Earth? where strongest? weakest?*

Earth's gravity keeps Moon in orbit  
otherwise it would fly away!

Moon's gravity pulls on every particle on Earth  
but: gravity force *decreases* with distance from Moon

- strongest pull at closest point “underneath” Moon
- weakest pull at farthest point, opposite sides
- medium pull at center of Earth

An illustrative simplified problem:  
three identical mass balls  $A, B, C$  in a line  
freely floating except for Moon's gravity



if Moon's gravity force *did not* change with distance  
but were the *same* on all three

*Q: how would the set of particles move relative to the Moon?*

*Q: how would the particles move relative to each other?*

In reality, Moon's gravity strength *decreases* with distance  
in this case:

*Q: how would the set of particles move relative to the Moon?*

26 *Q: how would the particles move relative to each other?*

If gravity were (unrealistically) *constant* with distance:

- all three particles feel the same force  
so the whole *ABC* system falls toward (or around) the Moon
- but they would *not* move relative to each other  
i.e., not pulled apart or together

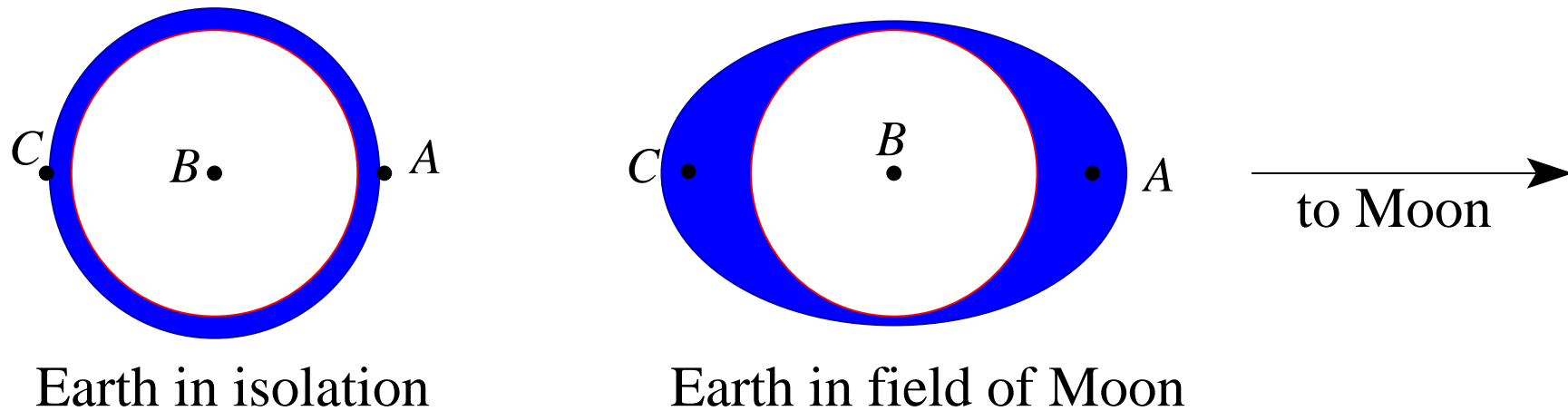
But gravity really *weakens* with distance:

- all particles still feel *some* attraction  
so the whole *ABC* system still falls  
toward (or around) the Moon, but...
- the farther particles feel progressively less force  
*A* pulled away from *B*, and *B* pulled away from *C*  
⇒ entire *ABC* system is internally **stretched!**  
relative to midpoint (*B*), edges pulled away!

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Given the realistic results

*Q: What is the Moon's effect on oceans?*



$A$  attracted **more** than average ( $B$ ),  $C$  less  
 → tides rise symmetrically along axis towards moon  
 i.e., tides high in region nearest **and farthest** from Moon  
 → high-tide to high-tide period **twice** daily

www: tide animation

28 What about the rocks on the Moon, and the Earth?  
*Q: how do changing gravity forces affect them?*

Tidal stresses on Moon → Moon surface constantly deformed

Deformed Moon non-spherical: tidal bulges

Earth gravity on bulges → twists Moon! (torque)

- if Moon spins faster than orbit → decreases Moon spin
- also decreases Moon orbit speed → moves Moon outward!  
measured! **the Moon moves away about 3.8 cm/year!**

repeated stretching/compression → friction, heating

dissipation → evolve to frictionless state:

reduces Moon spin rate until

$P_{\text{spin}} = P_{\text{orbit}}$ : corotation!

Note: may take long time!

complete for Moon, not for Earth!

∞ measured! Earth spin period increasing!

→ **day getting longer by about 0.0016 sec/century!**

ENOUGH ALREADY—TAKE A BREAK!

# The Statistical Sky: Solar System History from Lunar Craters

## Craters on the Moon and Earth

Compare pictures of Earth and Moon

www: global Earth

www: global Moon

*Q: similarities? differences?*

Note craters:

*Q: how are they made?*

*Q: why differences in Earth vs Moon cratering?*

...more than one reason!

## Crater Comparison

Craters made by explosions (including man-made events...)

larger explosion → larger crater size

in Solar System: explosions from asteroid/comet collisions

crater existence on a surface in Solar System

→ encodes info on bombardment history!

Why more craters on Moon?

Moon & Earth share same Solar System real estate

so exposed to same bombardment hazard, but...

★ Earth atmosphere burns up all but largest objects

★ Earth oceans hide 75% of surface

★ Earth erosion “quickly” erases damage

★ Earth volcanos hide some

★ Earth tectonic activity (plate motions) recycles surface

## Craters on the Earth

In fact: large objects *do* survive atmosphere, hit Earth surface, leave craters for millions of years today ~ 100 identified around the world!

www: terrestrial craters

Largest known crater: Chicxulub, Mexico

www: Chicxulub crater map

- 200 km across!
    - explosion = 40,000 × *entire global nuke arsenal!*
  - date: created 65 million years ago
  - also: dinosaurs wiped out 65 million years ago
- ⇒ probably due to explosion
- fires; tsunamis; dust → dark, cold for years

## Craters on the Moon

www: global Moon

www: Google Moon

*Q: what patterns do you notice—color, terrain, cratering?*

*Q: are craters equally numerous across Moonscape?*

*Q: what is this trying to tell us?*

# Lunar Craters and Solar System History

## Lighter-Colored Regions: “Highlands”

- ▷ rough, mountainous terrain
- ▷ high concentration of craters (Lunar DMV not on the job!)

## Darker-Colored Regions: Maria = “Seas”

- ▷ smooth plains
- ▷ many fewer craters (but still plenty of them)

Since cratering → bombardment history

fewer craters in Maria → **younger surface**

in fact: moonrocks from Maria are “basalt” = volcanic

⇒ Maria/seas are ancient lava flows → “repaved” part of Moon

⇒ Maria cratering gives bombardment rate since

⇒ early Solar System more collisions: a violent place!

⇒ in fact, Moon itself probably born in collision!

Mars-sized body + Earth → Moon!

ENOUGH ALREADY—TAKE A BREAK!

# The Physical Moon: Traveler's Tales, and Lessons from Moonrocks

## The Moon: Surface Features

★ **highlands**: lighter in color, heavily cratered

www: Apollo 17 in highlands (mountains made by impacts)

★ **maria** – “seas” (singular: mare): dark plains

www: Mare Imbrium large scale

www: maria/highlands comparison

smooth: fewer craters, made of volcanic rock *Q: how do we know?*

formed by lava flows

★ craters

cover surface

occur in all sizes, > 20km to microscopic

www: Mare Oriental

www: maria--overlapping craters

★ “soil” **regolith** = “rock blanket”

*transp: Niel Armstrong quote—right after first step*

www: footprint

www: Real Audio Armstrong--start at 3:35

dust, rock fragments

accumulated debris from many impacts

★ other tips for tourists:

- no atmosphere → no UV, X-ray protection

- slow rotation → long “days”

huge day/night temp diff: 370K vs 125 K

## The Moon: Origin

Earth & Moon similar in composition of crust, different in core

- Fission model (“sparated at birth”) :  
moon spun out from rapidly rotating earth
- Binary (“Sister”) model  
earth and moon formed together as binary system
- Capture model  
“vagabond” moon gravitationally captured by earth
- giant impact model  
Moon created in collision of Mars-sized object with early Earth

*Q: pros, cons of each?*

- Fission model ( “sparated at birth” ) :  
unlikely: ang mom problem—Earth wouldn’t rotate fast enough
- Binary ( “Sister” ) model  
unlikely: where do differences come from?
- Capture model  
unlikely: where do similarities come from?
- giant impact model  
“least unlikely” —accounts for both differences and similarities  
→ eary solar system a violent place!