

# The Quantitative Cosmos:

## Astronomy as a Launchpad for Math and Science

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webpage with images and links:

[http://www.astro.uiuc.edu/~bdfields/mste\\_aims06.html](http://www.astro.uiuc.edu/~bdfields/mste_aims06.html)

- ▶ The geometric sky: celestial sphere and star trails
- ▶ The dynamic sky: the Sun's apparent motion among the stars  
...in real time!
- ▶ Astronomical numbers and the architecture of the Universe  
...and why telescopes are time machines

# Introductions

Please allow me to introduce myself...

*Q: anybody teach any astro?*

*Q: if so, what? do you use math?*

Today's schtick based on my conviction:

**Astronomy** and **Math/Science teaching**  
are **two great tastes** that **taste great together!**

Astronomy can serve math & science by:  
illustrating, motivating, and enriching subjects you are  
*already teaching*

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Today's strategy: I'll provide the Astronomy  
but you know best how build your lessons!

# The Geometric Sky: Celestial Sphere

## Celestial Sphere and Star Trails

The Earth is a sphere (very nearly exactly)

equator: midway between poles

latitude: N-S from eq.

Chambana lat:  $40.11^\circ$  N

*Q: good geometry question—why need exactly two numbers (coordinates) to specify location on globe?*

## Earth Motion and Star Motions

Earth spins (rotates) once daily  
about axis: north pole–south pole  
*diagram: label poles*

but we don't notice our motion, and instead  
take “egoist view”: everything else is spinning!  
→ origin of rising and setting of Sun, planets, and stars

stars don't seem to move relative to each other (“fixed”)  
i.e., constellations don't morph!  
but our view from spinning Earth

↳ leads to apparent star motion in the night sky

*What is the observed motion of the stars?*

Imagine a dark night sky in Chambana  
and an expensive, magic machine:  
makes each star leave a “trail” behind as it moves

The Big Question:

*Q: if you could see all the trails made after one night  
what pattern would you see?*

- (a) circles
- (b) parallel lines
- (c) ovals
- (d) none of the above

science: to test prediction, do experiment!

o *Q: how to do the experiment?*

## The Experiment

fix a camera on a tripod, open lens and expose to night sky  
as each star moves, leaves “trail” on film

Many such images exist online:

www: `imagine star trails`

*Q: why do we get this pattern?*

*Q: what does it tell us? special points/regions?*

*Q: why do telescopes have motors on them?*

star trail pattern: concentric circles (really, arcs of circles)

appears as if stars fixed on a huge sphere

**celestial sphere** (celestial= “in the sky” )

rotates about celestial poles

*diagram: celestial sphere*

*Demo: celestial sphere globe*

Note: idea of “celestial sphere” is a tool:

helps visualize, understand; not meant as actual description  
of course earth spins and stars more or less fixed

∞ *Q: what is bright arc at hub, why not in S hemisphere?*

## Interlude: The Flat Sky

The celestial sphere and its motion are one example of a general problem in astronomy (and all science!)  
have to connect

- (a) what you can actually *observe/measure: data*
- (b) with what is “really” going on—*models/theory*

In astronomy: observe objects in sky  
can measure position on celestial sphere:  
2-number “address”  $\Leftrightarrow$  2-dimensional sky

but sky gives *no* direct information about *distance*  
→ observed sky flattens the 3-dimensional arrangements  
down to 2-D sphere projection: “cosmic roadkill”

Geometry problem: have to always go between

- ▷ observed 2-D sky view
- ▷ underlying 3-D space distribution

For stars

in 3-D space: distant stars, spinning earth

on 2-D sky as seen by earthling: spinning celestial sphere

Other “flattening” issues will come up later...

Exercise in geometry, spatial visualization.

## Mathematics of Star Trails

www: Star trails from mountaintop

*Q: at what latitude was this picture taken? How do you know?*

*Q: in this picture, how long was exposure?*

*Q: so what are the rules?*

*Q: what longest can see any star during night?*

*Q: do you see all stars for this length of time?*

*Q: how understand rising and setting of Sun? path on sky?*

*Q: where are stars during day?*

## Geometry—literally geometry—with star trails

angle from horizon  $\rightarrow$  cel. pole = latitude

star trail pix reveal global location of photographer

but don't really need trails  $\rightarrow$  just need to find pole

in N. hemisphere: N star will do; can find using big dipper  
(“pointer stars” in dipper lip show the way)

★ N star (N cel pole) “altitude” angle: latitude!  
go look tonight! then compare on your next trip  
to Alaska or Hawaii!

## Star Trail Motion

earth's rotation spins us under the stars  $360^\circ$  each day  
so star trail arc (angle  $\theta$ ) is *proportional* to exposure time  $t$

$$\frac{\theta}{360^\circ} = \frac{t}{24 \text{ hours}} \quad (1)$$

and thus we can solve for exposure time

$$t = 24 \text{ hours} \times \frac{\theta}{360^\circ} \quad (2)$$

e.g., an arc with  $\theta = 60^\circ$  (i.e., 1/6 of circle)

comes from an exposure of

$$t = 24 \text{ hrs} \times 60/360 = 4 \text{ hrs (i.e., 1/6 of a day)}$$

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about  $\theta \approx 180^\circ$  because...?

## Rising and Setting

Stars (cel sphere) rise in the East and set in the West

**really:** due to the Earth's spin carrying us underneath stars

**apparently:** due to cel sphere spin

But Sun, Moon, planets, etc. also “live” in celestial sphere  
they too rise in East, set in West

→ daily motion of astronomical objects set by  
nature of Earth's spin

But where are the Sun, Moon, planets in the celestial sphere  
i.e., what is their location among the stars?

in particular: are these objects always in the same place  
or do they move?

*Q: whaddaya think? How can you test this?*

ENOUGH ALREADY—TAKE A BREAK!

# The Dynamic Sun—In Real Time!

# The Sun's Passage Through the Constellations

Earth orbits (revolves) around Sun

**Period** = time for 1 complete orbit = 1 year

distance to Sun varies by  $\pm 2\%$  during year

→ orbit almost a perfect circle

In **3-D** space

Earth-Sun motion lies in a plane: the **ecliptic**

but due to orbit: Sun moves relative to stars

another case of 3-D action projected onto **2-D** sky

*Q: If could stand on Sun, what shape Earth's path on sky?*

*Q: From Earth, what shape is Sun's annual path on sky?*

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But can't see Sun, stars at same time

*Q: so how did the ancients know Sun's path relative to stars?*

annual Sun path on celestial sphere: *circle*=ecliptic  
(in fact, a great circle—not just pretty good!)

*K&C transp: Earth orbit + cel sphere + ecliptic*

Note: ecliptic is a path thru cel sphere  
constellations inside path: zodiac

How to find Sun's path?

old school: infer from knowledge of cel sphere

e.g., know which constellations near rising, setting Sun

new school: turn down Sun brightness in high-tech way

## SOHO and the Real-Time Sun

**SOHO**: SOLar and Heliospheric Observatory

NASA satellite dedicated to taking images of the Sun

www: SOHO page

SOHO's "coronagraph" blocks glare of the Sun's disk so we can see surrounding, less luminous, solar corona and the material it ejects

Side benefit: see background stars which appear to move relative to the Sun  
Can see this in real-time images!

www: SOHO coronagraph

much going on! *Q: what do you notice?*

You will notice various "burps", we'll get to these below, but for now just notice the stars.

*Q: what is Sun's angular speed wrt stars?*

calculate from image

*Q: how can we understand this?*

a diagram of Earth, Sun, stars helps

planets also move in same plane

occasionally pass in front or behind Sun

www: SOHO coronagraph images from May 16, 22

*Q: is planet on same side or opposite side of Sun as us?*

*Q: is planet moving in same or opposite direction as us wrt Sun?*

8 *Q: is planet moving around Sun faster or slower than us?*

*Q: based on this, guesses as to which planet is it?*

Planet can be seen: illuminated side is towards us

→ must be on opposite side of Sun

Planet moves opposite direction of stars on sky

→ in 3-D, going same direction as us around Sun

Planet “passes” Sun: must be going faster than us

⇒ *Mercury*

Other events on webpage:

Conjunction of Mercury, March 2003; of Jupiter, Aug 2003

note that in these images planets move

celestial sphere (the stars), but also move relative to Sun

and move at different angular speeds (degrees per day)

With a table of orbit distances, can work out speeds.

## Sunspots and Solar Rotation

SOHO webpage has real images of Sun

www: <http://sohowww.nascom.nasa.gov/data/latestimages.html>

can use movies of these to track sunspots

which are “magnetic storms” on Sun

From motion of spots across Sun’s disk:

→ calculate spin rate of Sun

can compare the Sun’s spin period (“solar day”)

to that of the Earth (the usual terrestrial day)

can also see that sunspots are transient: come and go,

over timescales of days (storms die out)

ENOUGH ALREADY—TAKE A BREAK!

# Astronomical Numbers

# Astronomical Numbers!

## The Cosmos Cries out for Scientific Notation

astronomy involves extremes of scales: time, distance, speed, ...  
quantifying these naturally requires astronomically large numbers

⇒ astronomy is a perfect illustration of, and motivation for,  
**scientific notation**

to give one example: **distances**

math and science give tools to survey the cosmos

for example, we want to know how far it is  
to the Moon, the Sun, other solar system locales  
then we'd like to know how far to the nearest star  
to the center of our Galaxy,  
to the nearest Galaxy,  
to the edge of the observable universe

even the closest astro-distances are enormous  
also deeper pattern: “hierarchy” of scales and structures f  
*Q: a useful and common idea in science—you know it even if the  
term is unfamiliar—anybody care to define?*

# The Hierarchical Universe

as we imagine “zooming out” our view, each scale:

- Sun/planet system
- stars in a galaxy
- galaxies in the universe

is much smaller than the next “rung on the ladder”  
and much larger than the preceding rung

for a better understanding, need to measure distances

Note: goes the other way too → hierarchy in micro-world

e.g., matter → molecules → atoms

atoms → electrons & nuclei; nuclei → protons & neutrons

protons & neutrons → quarks (→ ??)

## Geometry of the Solar System: Shape

Renaissance astronomers (e.g., Copernicus) found *relative* spatial arrangement of the planets (e.g., proportions) without needing to know how far the Sun and planets were in *absolute* terms like miles or km

How? *Geometry!*

Example: **Venus**

Data:

- Venus always rises  $\leq 3$  hours before sunrise
- Venus always sets  $\leq 3$  hours before sunset
- Venus never seen at or around midnight

*Q: what does this tell us about Venus' orbit around the Sun?*

*Q: what's going on in Venus' orbit when it rises the earliest, or when Venus set's the latest?*

Interpretation:

Venus is always seen close to the Sun

and never seen opposite the Sun (i.e., at midnight)

→ so Venus' orbit never takes it beyond Earth's orbit

...or even near the distance of Earth's orbit:

→ Venus' orbit lies *inside* that of Earth's

→ Venus is closer to the Sun than us!

*Diagram*

When Venus rises earliest or sets latest

it is making the largest Sun-Venus-Earth angle

“maximum elongation,” angle  $\alpha = 48^\circ$

*Diagram*

*Q: What is special about the EVS geometry at max elong?*

max elongation when Earth-Venus line barely touches  
(is *tangent* to) Venus' orbit circle

trigonometry: this means that the EVS triangle is a right triangle  
with max elongation  $\alpha$  the adjacent angle  
then: Earth-Sun distance  $a_E$  and  
Venus-Sun distance  $a_V$  are related by

$$a_V = a_E \sin \alpha \quad (3)$$

and so

$$\frac{a_V}{a_E} = \sin \alpha = 0.7 \quad (4)$$

Venus' orbital radius is 70% the size of ours!  
we have found proportions of our orbit shapes!

invent distance: “Astronomical Unit”: **AU**

1 AU =  $d_E$  = Earth-Sun distance

then Venus' orbit is 0.7 AU

can find: Mercury orbits at 0.4 AU

Jupiter at 5 AU

Pluto at 40 AU

*Q: but what don't we know still?*

## Geometry of the Solar System: Size

We don't know how far the planets are in "real units" i.e., in meters or km or miles

various tricks to measure

most precise modern method: radar

www: Jodell Bank radio observatory

wait for Venus to cross ("transit") disk of Sun

send radar signal, wait for it to bounce back

*diagram*

signal travels at speed of light:  $c = 299,792.458$  km/s

signal travels distance  $d = 2(a_E - a_V) = 0.56$  AU

measure time delay  $t = 300$  sec

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*Q: key last mathematical ingredient?*

distance related to travel time:  $d = ct$

so  $d = 299,000\text{km/s} \times 300\text{ s} = 90,000,000\text{ km}$ , and

$$0.6\text{ AU} = 90,000,000\text{ km}$$

$$1\text{ AU} = \frac{90,000,000\text{ km}}{0.6} = 149,597,870.691 \pm 0.030\text{km}$$

an astronomically large number! mind-bogglingly large!

but also inconvenient written this way...

e.g., Venus distance:  $0.7\text{ AU} = 108,208,926\text{ km}$

*yuck!*

Scientific notation to the rescue:

$$1\text{ AU} = 1.5 \times 10^8\text{ km} = 9.3 \times 10^7\text{ miles} = 93\text{ million miles} \quad (5)$$

∞ Note: from this, we can now get Earth's orbit speed

Q: *how?*

We go in circle, radius  $a = 1$  AU

in one cycle: time is  $t = 1$  yr  $= 3.15 \times 10^7$  sec

path traveled is  $d = 2\pi a$

so speed is

$$v = \frac{d}{t} = \frac{2\pi \text{ AU}}{1 \text{ yr}} \quad (6)$$

$$= \frac{1.5 \times 10^8 \text{ km}}{3.15 \times 10^7 \text{ sec}} \quad (7)$$

$$= 27 \text{ km/s} = 60,000 \text{ miles/hour!} \quad (8)$$

Note: we can also find other things  
on SOHO, look at movie of “proton showers”  
particles launched in flare, arrive at Earth about 2 hours later:  
speed

$$v = \frac{d}{t} = \frac{1 \text{ AU}}{120 \text{ min}} = \frac{1.5 \times 10^8 \text{ km}}{7200 \text{ s}} = 2 \times 10^4 \text{ km/s!} \quad (9)$$

for comparison, light moves at  $c = 3 \times 10^5 \text{ km/s}$   
so these protons move at  $v = 0.07c$ : 7% of speed of light!  
→ the Sun is somehow a particle accelerator, like Fermilab!

## Distances to Stars: Parallax

to understand our Sun's place among the stars  
and to show the Sun *is* a star and the stars are Suns  
need to find **distances** to stars

→ a challenging problem: no Galactic DOT to post roadsigns!

The “gold standard” – **parallax**

You know and use this all the time:

it's how your eyes & brain perceive depth

Stupid parallax tricks:

- close/cover one eye
- hold out your arm, cover my face with your thumb
- keep your arm still, but switch eyes

*Q: what do you see? why?*

*Q: is the effect smaller or larger when you move your arm closer?*

*Q: how could we use this effect for astronomy?*

## Parallax and Stars

Parallax: shifted viewpoint → foreground objects appear to move relative to distant background object  
closer objects → larger apparent shift

Everyday: two eyes → two viewpoints → depth perception  
but doesn't work for objects very far away

Astronomers get two viewpoints due to Earth orbit:  
take observations 6 months apart  
*diagram*

*Q: how does parallax and trigonometry give star distance?*

## A Trigonometric Distance

Data: measure star shift over 6 month interval  
find “parallax angle”  $p$ , half of total shift on sky

Notice right triangle:

Sun-star distance  $d$  and Sun-Earth distance  $a = 1$  AU related by

$$a = d \tan p \quad (10)$$

But we know  $a$  and  $p$  so we can solve for

$$d = \frac{a}{\tan p} \quad (11)$$

in fact, typical shift  $p$  is **tiny**

**largest** shifts:  $\approx 1$  arc second =  $1/60$  arc minute =  $1/3600$  deg

$\infty$  so  $\tan p$  also tiny: “skinny triangle”

$\rightarrow$  star distances  $d \gg a$ : very far compared to 1 AU

# Star Distances

Define distance unit **parsec**:

$$1 \text{ parsec} = \text{distance for } p = 1 \text{ arc sec} \quad (12)$$

$$= 2 \times 10^5 \text{ AU} \quad (13)$$

$$= 3.15 \times 10^{13} \text{ km} \quad (14)$$

$$= 1.92 \times 10^{13} \text{ miles} \quad (15)$$

way more mind-bogglingly large than an AU

but note comparison with AU: gives sense of *relative* scale

Parallax measurements have been used to map the nearest stars

- closest neighbor:  $\alpha$  Centauri (really a 3-star system)

$$p_{\alpha\text{Cen}} = 0.6 \text{ arc sec} \rightarrow d_{\alpha\text{Cen}} = 1.3 \text{ parsec}$$

- average spacing between stars in our Galaxy: about 1 pc  
 $\Rightarrow$  a parsec is typical distance between stars (in a galaxy)

## Algebra in the Sky: Inverse Proportionality

Exact expression connecting distance and parallax:  $d = a / \tan p$   
but: in practice  $p$  is *tiny*!

for small angles (skinny triangles), an excellent approximation is

$$\tan \theta \approx \theta_{\text{rad}} \quad \text{try it!} \quad (16)$$

where  $\theta_{\text{rad}}$  is the angle in **radians**

i.e.,  $\theta_{\text{rad}} = \pi \theta_{\text{deg}} / 180^\circ$

Using this, the parallax equation simplifies to an *inverse proportionality*:

$$d \approx \frac{a}{p_{\text{rad}}} = \frac{1 \text{ parsec}}{p_{\text{arcsec}}} \propto \frac{1}{p} \quad (17)$$

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*Demo:* can verify trend with your thumb

insight from  $d \propto 1/p$

*Q: why did the ancient Greeks use parallax to argue against the Earth's moving?*

*Q: what are the limitations of the parallax technique—when will it fail to produce measurable results?*

*Q: what do you do then?*

## Telescopes are Time Machines

key idea: the speed of light  $c$  is large but **finite**  
i.e., not infinite, so light takes *time* to travel  
usually don't notice: crosses a 10 meter room in  
just  $t = d/c \approx 3 \times 10^{-7}$  sec!

but stars are so far away that even light  
takes a long time to make the journey  
→ we see stars not as they **are** now,  
but as they **were** when the light began its trip

and since  $t = d/c \propto d$ : the farther away, the longer ago!

→ *telescopes are time machines*

in 1 year, light travels

$$d = c \times 1 \text{ year} = 10^{13} \text{ km} = 0.3 \text{ parsecs} \quad (18)$$

so we see the **nearest** star as it **was** 4 yr ago!

...they don't know the Sox finally won the Series!

For more distant stars, light takes longer

→ most of the stars in the sky shine with light  
which began its trip before we were born!

the most distant stars you can see with your eyes  
are about 1000 parsecs away: light travelled 3000 years!  
began its journey in Y-1K!

And using telescopes, we can see further, to other galaxies  
and across the universe

most distant galaxies are seen billions of years ago!  
cosmic history is revealed by looking far away!

## Bonus: More Examples of Math in the Sky

Geometry: Planet Orbits

orbits are not perfect circles, but **ellipses**

Kepler's 1st Law: each orbit is an ellipse

with the Sun at one focus (nothing at the other!)

ellipse anatomy: definition, foci, semimajor axis  $a$ , eccentricity  $e$

Quadratic and Cubic Powers: Planet Motion

we saw: planets orbit at different distances from Sun

also true: orbit times—period  $P$ —differ

qualitative trend: farther orbit  $\rightarrow$  longer  $P$

quantitative result: Kepler's 3rd Law

$$P_{\text{yr}}^2 = a_{\text{AU}}^3 \quad (19)$$

44 can verify: works for Earth

with calculator can verify for all planets

but even without calculator:

*Q: is Mercury's "year"  $P$  longer or shorter than ours?*

*Q: is Jupiter's "year"  $P$  longer or shorter than ours?*